Light Bullet Modes in Self-Induced-Transparency Media with Refractive Index Modulation

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We predict the existence of a new type of spatiotemporal solitons ("light bullets") in two-dimensional self-induced-transparency media with refractive index modulation in the direction transverse to that of pulse propagation. These self-localized guided modes are found in an approximate analytical form, their existence and stability being confirmed by numerical simulations, and may have advantageous properties for signal transmission.

"Light bullets" are multi-dimensional solitons that are localized in both space and time. In the last decade they have been theoretically investigated in various nonlinear optical media [1, 2, 3, 4, 5, 6, 7, 8, 9, 10], and the first experimental observation of a quasi two-dimensional (2D) bullet was recently reported [11]. A promising candidate for the observation of fully 2D and 3D light bullets is a self-induced-transparency (SIT) medium. SIT involves undistorted and unattenuated propagation of an electromagnetic pulse in a medium consisting of near-resonant two-level atoms, irrespective of the carrier-frequency detuning from the resonance [12, 13]. As early as 1984, simulations had shown self-focusing of spatiotemporal pulses in a SIT medium into a quasi-stable vibrating object [14], thus hinting at the possible existence of "bullets". In recent works [9, 10], we have predicted that both uniform 2D and 3D SIT media [9] and SIT media embedded in a Bragg grating [10] can support stable light bullets (LBs). Here we complement these investigations with the case of SIT media with refractive index (RI) modulation in the direction *transverse* to that of pulse propagation. We show that such a structure acts as a unique nonlinear waveguide, which transversely guides (confines) a new type of LBs, whereas other pulses are dispersed and diffracted [15]. Following the discovery of SIT soliton propagation in erbium-doped resonant fiber waveguides [16], a demonstration of the existence of light bullet guided modes may lead to new possibilities for optical signal processing.

Our starting point is a 2D SIT medium with a spatially-varying refractive index $n(z,x)$, which is described by the lossless Maxwell-Bloch equations [17],

$$-i\mathcal{E}_{xx} + n^2\mathcal{E}_\tau + \mathcal{E}_z + i(1-n^2)\mathcal{E} - \mathcal{P} = 0, \quad (1a)$$

$$\mathcal{P}_\tau - \mathcal{E}W = 0, \quad (1b)$$

$$W_\tau + \frac{1}{2}(\mathcal{E}^*\mathcal{P} + \mathcal{P}^*\mathcal{E}) = 0. \quad (1c)$$

Here $\mathcal{E}$ and $\mathcal{P}$ are the slowly varying amplitudes of the electric field and polarization, W is the inversion, z and x are the longitudinal and transverse coordinates (in units of the resonant-absorption length), and τ is time (in units of the input pulse duration). The Fresnel number F, which governs the transverse diffraction in 2D and 3D propagation, has been incorporated in x and the detuning $\Delta\Omega$ of the carrier frequency ω_0 from the central atomic-resonance frequency was absorbed in $\mathcal{E}$ and $\mathcal{P}$. They can be reintroduced into Eqs. (1) by the transformation $\mathcal{E}(\tau,z,x) \to \mathcal{E}(\tau,z,x)\exp(-i\Delta\Omega\,\tau)$, $\mathcal{P}(\tau,z,x) \to \mathcal{P}(\tau,z,x)\exp(-i\Delta\Omega\,\tau)$, and $x \to F^{-1/2}x$. We have neglected polarization dephasing and inversion decay by assuming pulse durations that are short on the time scale of these relaxation processes. Equations (1) are then compatible with the local constraint $|\mathcal{P}|^2 + W^2 = 1$, which corresponds to Bloch-vector conservation [17]. In the absence of the x-dependence and for $n(z,x) = 1$, Eq. (1a) reduces to the sine-Gordon (SG) equation which has the soliton solution $\mathcal{E}(\tau,z) = 2\alpha\,\mathrm{sech}(\alpha\tau - z/\alpha + \Theta_0)$, where α and Θ_0 are real parameters.

Our aim is now to investigate whether there exist stable light bullet solutions of (1) for a specified RI profile $n(x)$. The physical idea behind this search is well-known: a specific transverse modulation of the refractive index can compensate for the transverse diffraction of a specific pulse (described by the first term in (1a)), and thereby form a unique nonlinear waveguide confining that pulse whereas others may be diffracted. This phenomenon of guidance by transverse confinement has been demonstrated many times in waveguide theory and fiber optics [18], but as far as we know guidance of LBs has not yet been investigated. Since light bullets are not only spatially but also temporaly localized, the possibility of "light bullet guided modes" is both interesting from a fundamental point of view and may also open new possibilities in signal transmission.

We search for a stable solution of Eqs. (1) which (i) is simultaneously localized in time and in the transverse direction, (ii) reduces to the SG soliton in 1D, and (iii)

whose transverse diffraction is compensated by a suitably chosen RI modulation. The first criterion is satisfied for an electric field amplitude of the variable-separated form $\mathcal{E}(\tau, z, x) = \mathcal{E}_1(\tau, z)\, \mathcal{E}_2(x)$. Using this form, we find an approximate analytical ansatz solution of (1) which meets all three requirements:

$$\mathcal{E} = \pm 2\,\alpha\,\mathrm{sech}\,\Theta(\tau, z)\,\mathrm{sech}(\beta x)\,\exp(i\phi), \quad (2a)$$

$$\mathcal{P} = \pm 2\,\mathrm{sech}\,\Theta(\tau, z)\,\tanh\Theta(\tau, z)\,\mathrm{sech}(\beta x)\,\exp(i\phi) \quad (2b)$$

$$W = [1 - 4\,\mathrm{sech}^2\Theta(\tau, z)\,\tanh^2\Theta(\tau, z)\mathrm{sech}^2(\beta x)]^{1/2} \quad (2c)$$

with

$$\Theta(\tau, z) \equiv \alpha(\tau - z) - z/\alpha + \Theta_0, \quad (2d)$$

in the presence of the RI profile, see Fig. 1,

$$n^2(x) = \begin{cases} 1 - \beta^2 \left[1 - 2\mathrm{sech}^2(\beta x)\right], & \text{for } |x| \leq x_{\max} \\ 1, & \text{for } |x| \geq x_{\max} \end{cases} \quad (2e)$$

Here $x_{\max} \equiv \mathrm{arcsech}(1/\sqrt{2})/\beta$, and α, β, ϕ and Θ_0 are real constants. The ansatz (2) satisfies Eq. (1a) exactly, while Eqs. (1b) and (1c) are satisfied in the limit $\beta|x| \ll 1$. Numerical results displayed below verify that Eqs. (2) indeed approximate a solution to Eqs. (1) to a high accuracy. The field (2a) represents a LB traveling in the z-direction with a velocity $v = \alpha^2/(\alpha^2 + 1)$ and reduces to the SG soliton in the limit $\beta \to 0$, when the transverse guidance disappears. Hence this approximate "light bullet" solution is in fact a *nonlinear multi-dimensional guided mode*. Just as the 1D SG soliton it propagates undistorted and unattenuated and satisfies the SIT area-theorem [12] $\int_{-\infty}^{\infty} d\tau |\mathcal{E}(\tau, z, x = 0)| = \pm 2\pi$. The LB mode (2) is unrelated to the LB found in Ref. [9] in a uniform SIT medium; in fact, it can be proven that light bullets of variable separated form as in (2) do not exist in a uniform SIT medium [9], nor in a SIT medium embedded in a Bragg grating in which the refractive index varies in the longitudinal direction [10]. Eq. (2) represents a new type of nonlinear guided mode.

In order to test the accuracy and stability of the ansatz (2), we have performed numerical simulations. Figure 2 shows the electric field at $z = 1000$, generated by direct simulation [19] of the 2D SIT equations (1) using Eqs. (2) as an initial configuration at $z = 0$. To a very good accuracy (with a deviation $< 1\%$), the result of the evolution over this propagation distance, which is much larger than the corresponding diffraction length, still coincides with the initial field configuration. The corresponding polarization distribution (not shown) has the shape of a double-peaked bullet in τ and decays in a similar way as the electric field, which is a characteristic property of SIT [17]. Also the inversion is localized in both τ and x, taking at infinity the value -1, corresponding to the atoms in the ground state. We have checked that no instability of the LB solution sets in as long as the simulations were run, up to $z \sim 10^4$. In addition, we have gathered
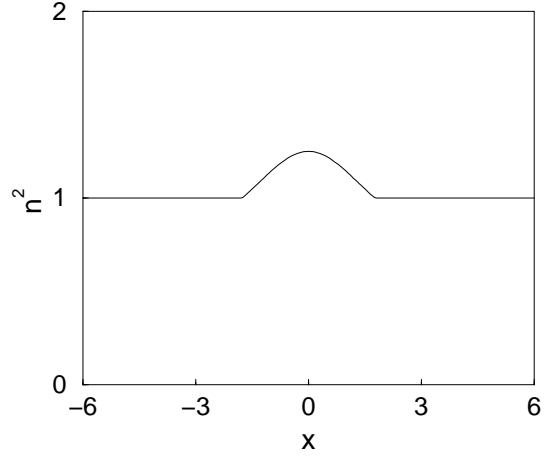

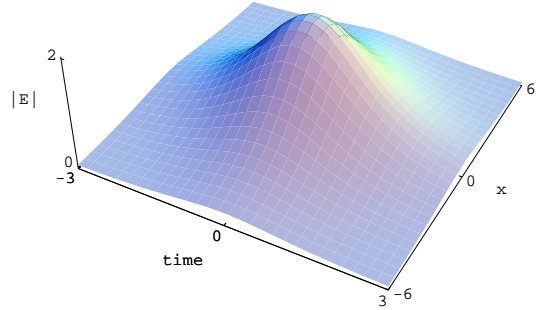FIG. 1: The profile of the guiding structure corresponding to the modulated refractive index (2e) with $\beta = 0.5$.

FIG. 2: The electric field inside the 2D "light bullet", $|\mathcal{E}|$, in a SIT medium with a transversely-modulated refractive index as per Eq. (2e), vs. time τ and transverse coordinate x after having propagated the distance $z = 1000$. Parameters used correspond to $\alpha = 1$, $\beta = 0.5$, $\phi = 0$ and $\Theta_0 = 1000$.

numerical evidence that the modulation (2e) uniquely confines the LB mode (2), whereas other pulses experience transverse diffraction. Starting e.g. with the light bullet ansatz from Ref. [9] for a uniform SIT medium, we found that they diffract after propagating a distance $x \sim 10$ [20]. An accurate determination of the degree of uniqueness with which a particular profile corresponds to a particular LB guided mode requires further investigations though. Similarly, it is interesting to investigate how different RI modulations may guide other types of LB modes [21, 22].

Experimentally, nanolithography techniques allow for fabrication of dielectric structures with layer thicknesses $\sim$ a few atomic layers [23], and the study of light-matter interactions in such structures has developed into a vast research area [24]. In particular, erbium-doped resonant fiber waveguides, which were developed about a decade ago and in which SIT has been observed [16],

form interesting candidates for opening new fields in optical communications. Generation of light bullet modes in transversely modulated SIT structures presents a new experimental challenge in this field. We briefly discuss here experimental conditions under which the guided LB modes predicted above may be observed. A suitable SIT medium consists of rare-earth ions embedded in a dielectric structure, with a typical density $\sim 10^{16}$ cm^{-3} [23] and radiative relaxation time (at cryogenic temperatures) ~ 100 ns [25]. The incident optical pulse, of duration $\tau_p < 0.1$ ns, should have uniform transverse intensity and, for the transverse diffraction to be strong enough, one needs $\alpha_{\rm eff} d^2/\lambda_0 < 1$ [26], where $\alpha_{\rm eff}$, λ_0 and d are the inverse resonant-absorption length, carrier wavelength, and pulse diameter resp. For $\alpha_{\rm eff} \sim 10^4$ m^{-1} and $\lambda_0 \sim 10^{-4}$ m, one thus requires $d < 10^{-4}$ m, which implies that the transverse medium size $L_x \sim 1 - 10\,\mu$m. The parameter β in Eq. (2) is determined by the transverse component of the wavevector κ_x and we find $\kappa_x L_x \sim 0.01 - 0.1$, which is in accordance with the physically relevant regime of $n(x) \geq 1$. The parameter α, which determines the amplitude of the bullet and its localization in τ and z, is given by $\alpha = \sqrt{v\alpha_{\rm eff}\tau_p(1 - v\alpha_{\rm eff}\tau_p)}$, and can thus be controlled by the incident pulse duration and velocity. For rare-earth media, one typically has $\alpha \sim 0.1 - 10$. Embedding this medium into a set of 10-100 transverse layers, each having a thickness on the order of the wavelength, and RI varying by $\sim 1\%$ from layer to layer [18], completes the setup for the observation of the 2D guided-LB modes. These are then expected to be localized on a time scale ~ 100 ps and transverse length scale $\sim 1\,\mu$m.

To conclude, we have proposed and studied the novel regime of light bullets in SIT media embedded in transversely modulated structures. Such a structure is predicted to allow one to produce nonlinear guided light bullet modes of a specific and fully controllable shape, while causing other pulses to disperse via transverse diffraction. Because of their simultaneous spatial and temporal localization, these LB modes may open new possibilities of signal transmission in nonlinear waveguides and we hope they will trigger new experiments.

[1] Y. Silberberg, Opt. Lett. **15**, 1282-1284 (1990).
[2] K. Hayata and M. Koshiba, Phys. Rev. Lett. **71**, 3275-3278 (1993); D.V. Skryabin and W.J. Firth, Opt. Comm. **148**, 79-84 (1998); D. Mihalache et al., Opt. Comm. **159**, 129-138 (1999); A. Desyatnikov et al., Phys. Rev. E **61**, 3107-3113 (2000).
[3] B.A. Malomed et al., Phys. Rev. E **56**, 4725-4735 (1997).
[4] D. Mihalache et al., Opt. Comm. **152**, 365-370 (1998).
[5] H. He and P. Drummond, Phys. Rev. E **58**, 5025-5046 (1998).
[6] K.V. Kheruntsyan and P.D. Drummond, Phys. Rev A **58**, 2488-2499 (1998); ibid. R2676-R2679 (1998).
[7] D.J. Frantzeskakis et al, Phys. Lett. A **248**, 203-207 (1998).
[8] G. Gottwald et al., Phys. Lett. A **248**, 208-218 (1998).
[9] M. Blaauboer, B.A. Malomed, and G. Kurizki, Phys. Rev. Lett. **84**, 1906-1909 (2000).
[10] M. Blaauboer, G. Kurizki, and B.A. Malomed, Phys. Rev. E **62**, R57-R59 (2000).
[11] X. Liu, L.J. Qian, and F.W. Wise, Phys. Rev. Lett. **82**, 4631-4634 (1999).
[12] S.L. McCall and E.L. Hahn, Phys. Rev. Lett. **18**, 408-411 (1967); G.L. Lamb, Rev. Mod. Phys. **43**, 99-124 (1971).
[13] A.A. Maimistov et al., Phys. Rep. **191**, 1-108 (1990).
[14] P.D. Drummond, Opt. Comm. **49**, 219-223 (1984).
[15] A somewhat similar situation was considered by S.Raghavan and G.P. Agrawal, Opt. Comm. **180**, 377-328 (2000), who predict that parabolic transverse modulation of the RI allows for stabilization of LBs in a Kerr medium.
[16] M. Nakazawa et al., Phys. Rev. A **45**, R23-R26 (1992); ibid. R2682-R2685 (1992).
[17] A.C. Newell and J.V. Moloney, Nonlinear Optics (Addison-Wesley, Redwood City, 1992).
[18] See e.g. L. Levi, Applied Optics, (Wiley, New York, 1980), Ch. 13.
[19] See the numerical scheme proposed in P.D. Drummond, Comput. Phys. Comm. **29**, 211-225 (1983).
[20] These bullets can be guided in the presence of a different refractive index modulation, given by $n(x) = 1 - \frac{1}{2}C^2(-1 + \frac{1}{2}{\rm sech}^2\Theta_1 + \frac{1}{2}{\rm sech}^2\Theta_2 + \tanh\Theta_1\tanh\Theta_2)$, with C, Θ_1 and Θ_2 as defined in Ref. [9].
[21] As an example of another type of 2D guided light-bullet modes, one can find a family of LBs corresponding to $\alpha = \infty$ in Eq. (2d). Their velocity, given by the expression $v = \alpha^2/(\alpha^2 + 1)$, takes the maximum possible value, $v = 1$. These solutions are obtained by substituting a plane-wave (in z) ansatz $\mathcal{E}(\tau,z,x) = \mathcal{E}(\tau,x)\exp(-ikz)$, $\mathcal{P}(\tau,z,x) = \mathcal{P}(\tau,x)\exp(-ikz)$, and $W(\tau,z,x) = W(\tau,x)$, with an arbitrary real constant k, into Eqs. (1). The equation for the field then becomes

$$-i\mathcal{E}_{xx} + n^2\mathcal{E}_\tau + ik\mathcal{E} + i(1 - n^2)\mathcal{E} - \mathcal{P} = 0, \qquad (3)$$

with the equations for $\mathcal{P}$ and W given by (1b) and (1c). If the RI in the medium is modulated as

$$n^2(x) = 1 - \beta^2\left[\tanh^2(\beta x) - {\rm sech}^2(\beta x)\right] + k\beta,$$

the LB solution to Eq. (3) can be approximated by the expression (2), with $\Theta(\tau,z)$ replaced by $\tau + \Theta_0$. Thus, these solutions are localized in τ and x, but at a fixed τ they are not localized in z.
[22] A similar guided LB as (2) can be found in a three-dimensional SIT medium embedded in a cylindrical waveguiding structure. The medium is described by Eqs. (1) with $\mathcal{E}_{xx} \to \mathcal{E}_{rr} + \frac{1}{r}\mathcal{E}_r$, where $r \equiv \sqrt{x^2 + y^2}$ is the transverse radial coordinate. Searching for an axisymmetric solution of these 3D equations, we arrive at an approximation of the same form as Eqs. (2), but with x replaced by r, and a corresponding cylindrical RI mod-

ulation

$$n^2(r) = 1 - \beta \left[\beta(1 - 2\mathrm{sech}^2(\beta r)) - \frac{\tanh(\beta r)}{r} \right],$$

for $|\beta r| \ll 1$. Comparison with results of simulations of the cylindrically symmetric 3D equations, using the analytical approximation as an initial ansatz, again shows good agreement (with a deviation $< 2\%$). In practice, however, such a 3D guided LB and waveguiding structure are probably much harder to realize than their 2D counterparts.

[23] G. Khitrova *et al.*, Rev. Mod. Phys. **71**, 1591-1639 (1999).
[24] See the *Photonic Band-Gap Bibliography,* compiled by J.P. Dowling, H. Everitt, and E. Yablonovitch, at http://home.earthlink.net/~jpdowling/pbgbib.html
[25] C. Greiner et al., Phys. Rev. A **60**, R2657-R2660 (1999).
[26] R.E. Slusher in: *Progress in Optics* vol. 12, ed. by E. Wolf (North-Holland, Amsterdam, 1974).